

St. Joseph's Catholic Primary School

Fluency Progression Document

The NCETM set out 5 main ideas for mastery: including coherence, representations, variation, mathematical thinking and fluency. They state that fluency is more than just memorisation of a single procedure or collection of facts. Quick and efficient recall of facts and procedures is important in order for children to keep track of sub-problems, think strategically and solve problems. Fluency also demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections.

These are broken down below into skills which each year group should focus on and children should be encouraged to regularly use and compare the different methods. Word aware and terminology is highly important in supporting the children's understanding of these different methods. Presenting the terminology on the working wall and modeling the terminology is paramount- understanding what is expected in each method and the steps they need to follow.

Definitions

Arithmetic equations and their parts

Addition equation

$$\begin{array}{r} 2 \quad 5 \\ + 1 \quad 1 \\ \hline 3 \quad 6 \end{array}$$

← addend
← addend
← sum

Subtraction equation

$$\begin{array}{r} 2 \quad 5 \\ - 1 \quad 1 \\ \hline 1 \quad 4 \end{array}$$

← minuend
← subtrahend
← difference

Multiplication equation

$$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$$

← factor
← factor
← product

Division equation

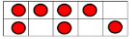
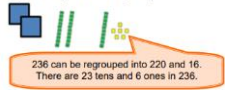
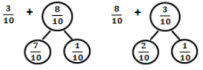
$$\begin{array}{r} \text{divisor} \\ \downarrow \\ \text{dividend} \quad 12 \div 4 = 3 \quad \text{quotient} \\ \downarrow \\ \text{divisor} \\ \downarrow \\ \text{dividend} \end{array}$$

← quotient
← divisor
← dividend

The Commutative Law

The Commutative Law says that when you add or multiply numbers, you get the same answer if you swap the numbers round.

$$\begin{array}{c} \text{6 dots} + \text{3 dots} = \text{9 dots} \\ 6 + 3 = 9 = 3 + 6 \\ \text{4 dots} \times \text{2 dots} = \text{8 dots} \\ 4 \times 2 = 8 = 2 \times 4 \end{array}$$

Fluency skill	Reception	Year 1	Year 2	Year 3	Year 4	Years 5 and 6
Subitising	Recognise groups up to 5 without the need to count Identify 5 and use as a benchmark number e.g. Numbers 6 and 7 where 5 is the benchmark i.e. 6 is 1 more than 5 using five frames and additional counters.  Ensure transference to fingers.  7 fingers can be shown by 5 fingers and 2 more fingers. Identify numbers within a whole set e.g.  I can see 3 and 3 and 1. I can see 4 and 1 and 1 and 1. I can see 4 and 3.	Recognise groups (up to 5) without the need to count and use this to identify numbers up to 10  I can see three and three and one makes seven. Four and one and one and one makes seven.				
Regrouping (partitioning)	Use 5 as a benchmark number Think 5 Where the whole is 6 or 7 and one of the parts is 5 (to secure benchmark from 5).  Six is one more than five. Five is one fewer than six. Understand a whole = part and part	'Think 10' – able to partition a number into 10 and some	Regroup two-digit numbers flexibly and in multiple ways e.g.	Regroup three-digit numbers flexibly and in multiple ways e.g. Using part part whole models, regroup 3-digit integers flexibly and in multiple ways.  236 can be regrouped into 220 and 16. There are 23 tens and 6 ones in 236. Regroup for addition to allow bridging through	Regroup four-digit numbers flexibly and in multiple ways Regroup tenths and hundredths flexibly and	Use regrouping as a valid method in a multi-step problem e.g., $\frac{3}{10} + \frac{4}{5} =$  $\frac{3}{10} + \frac{8}{10} =$ Rehearse regrouping either addend to make 1s and some more. 

Four and two make six. Two more than four is six. There is a four and two in six.

Identify a group as a unit e.g., a unit is 2 teddies

more e.g.

I made 14 into 10 and 4 more. There is 1 ten and 4 ones in the number 14. Here is the ten and here are the 4 more.

Able to regroup a whole up to 10 into different parts and understand the commutativity of this

Able to use 10 as a benchmark for regrouping for addition up to 20 (think "10 and some more") e.g.,

Regrouping numbers to 20 leading to 'think 10 for addition'. Pupils should experience regrouping either addend.

8 + 6 =

can become

or

8 + 2 + 4 = 10 + 4 =

6 + 4 + 4 = 10 + 4 =

I know that eight and two make ten. Four and two make six. So 8 + 6 can become 8 + 2 + 4.

I know that six and four make ten. Four and four make eight. So 8 + 6 can become 6 + 4 + 4.

Able to use 10 as a benchmark for regrouping for subtraction up to 20, using either the

36 - 10 =

36 - 10 = 26

Able to 'think 10' for addition e.g.,

$\square = 17 + 8$

a) Regrouping the second addend

17 + 8 =

17 + 3 + 5 =

20 + 5 = 25

b) Regrouping the first addend

17 + 8 =

12 + 8 + 5 =

20 + 5 = 25

Able to 'think 10' for subtraction e.g.,

Exploring that either the minuend or the subtrahend can be regrouped.

25 - 13 =

Regrouping the minuend (two examples). Taking from a multiple of ten or taking to a multiple of ten.

25 - 13 =

20 - 13 + 5 = 7 + 5 = 12

25 - 13 =

23 - 13 + 2 = 10 + 2 = 12

Regrouping the subtrahend - normally to a multiple of ten.

25 - 13 =

25 - 5 = 20 - 8 = 12

10 and 100. Work flexibly and reason about the most efficient methods.

Regroup for subtraction to allow bridging through 10 and 100.

In multiplication, regroup to the 10-benchmark working flexibly to regroup either the multiplier or the multiplicand e.g., 12×5

$12 \times 5 = 10 \times 5 + 2 \times 5$

in multiple ways e.g.,

I can see three tenths and four hundredths.

I can see that in the number 25.36, there are 2 tens, 5 ones, 3 tenths and 6 hundredths.

We can also say there are 253 tenths and 6 hundredths in my number.

In addition, regroup to 'think 100' with more complex 3-digit addends e.g.,

376 + 158

376 + 24 = 400

376 + 24 = 400

In subtraction, regroup to 'think 100' with more complex 3-digit numbers

Use regrouping strategies to fractional part wholes, as well as other measures such as time and money e.g.

For example: $\frac{4}{7} + \frac{3}{7} =$

Both addends can be regrouped using complements to 1 and 'some more'.

$\frac{4}{7} + \frac{3}{7} = 1 \frac{1}{7}$

Regroup for multiplication in a variety of ways e.g., 24×3

$24 \times 3 =$

$24 \times 3 = 72$

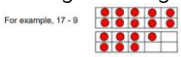
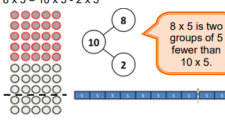
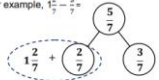
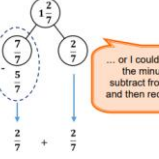
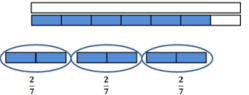
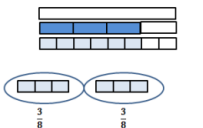
I know that $10 \times 3.4 = 34$. Then I can halve 34 to find 5 groups of 3.4 which is 17. After that, I have to recombine the products. This equals 51.

Regroup for division in a variety of ways e.g., $72 \div 3$

$72 \div 3 =$

$72 \div 3 = 24$

Regroup for division of fractions by whole

		minuend or the subtrahend, bridging through ten e.g. For example, $17 - 9$  Regrouping the subtrahend  Regrouping the minuend  Nine can be regrouped into 7 and 2. I can take 7 from 17 to leave 10 and then I can use my number bonds to take away 2 more. Seventeen can be regrouped into 10 and 7. Then I can use my number bonds to take 9 from 10. I'm left with 1. Then I add one to seven.	'Think 5' for multiplication and division e.g. $6 \times 5 = 5 \times 5 + 5 \times 1$  5 x 5 and one more group of 5 is equal to 6 x 5. Think 10 for multiplication and division $8 \times 5 = 10 \times 5 - 2 \times 5$  8 x 5 is two groups of 5 fewer than 10 x 5.		For example, $1\frac{2}{7} - \frac{5}{7}$  I can regroup the subtrahend $\frac{5}{7}$ into $\frac{2}{7}$ and $\frac{3}{7}$. Then I can take away the $\frac{2}{7}$ leaving $\frac{1}{7}$ or 1 and finally take away $\frac{1}{7}$.  ... or I could regroup the minuend, subtract from the 1 and then recombine. Use regrouping methods to find unknown products in multiplication e.g. 7×8 I can find 7×8 in lots of ways if I didn't know it. I can try $7 \times 4 \times 2$ or $8 \times 8 - 8$.	numbers e.g., I know that $12 \div 3$ can be thought of as 'If I share 12 equally between 3 groups, how many in each group?' So $\frac{6}{7} \div 3$ can be thought of as, 'If I share $\frac{6}{7}$ equally between 3 groups, how many in each group?' $\frac{6}{7} \div 3$  $\frac{3}{4} \div 2$  $\frac{3}{4} \div 2$ can be understood as: 'If I share $\frac{3}{4}$ equally between 2 groups, how many in each group?' Regroup fractions for multiplication (focus on this understanding before applying the rule) e.g., I know that 3×4 could mean 3 groups of 4. So $\frac{1}{2} \times \frac{1}{4}$ means half a group of $\frac{1}{4}$. $\frac{1}{2} \times \frac{1}{4} =$  When we find half of any number, we divide it by two. The blue part has a value of $\frac{1}{4}$. When I halve it, it makes $\frac{1}{8}$.
Counting on and back	Be able to count fluently. To do this, children need to have mastered:	Count on to find the total and difference	Count on and back from any two-digit number and notice what	Count on and back from any three-digit number and notice	Count on and back from any four-digit number and notice what changes and what	

The one-one principle – each object is counted once and given one counting tag

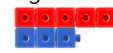
The stable-order principle – we use the words in the same order

The abstraction principle – counting can apply to objects which are not tangible e.g., number of claps

The order-irrelevance principle – it doesn't matter which order we count objects in

Be able to identify the largest number and count on from it

e.g.



Three and two more equals five.
Two fewer than five equals three.

Count on to find the total and difference and link this knowledge to fact families e.g.,



$14 + \square = 16$
 $\square + 14 = 16$
 $16 - 14 = \square$
 $16 - \square = 14$

14 and 2 more equals 16.
2 fewer than 16 equals 14.

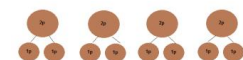
Be able to skip count in a variety of ways to promote the 2x tables and doubles

e.g.,

Counting groups of objects with two hands (drawing out understanding of doubles).



To include opportunities to count in 2s in several ways

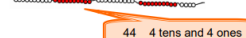


Be able to skip count in 5s and 10s in a variety of ways, forwards, backwards

changes and doesn't e.g.



34 3 tens and 4 ones



44 4 tens and 4 ones



46 4 tens and 6 ones



36 3 tens and 6 ones



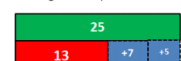
26 2 tens and 6 ones

I can see that the tens are changing but the ones are staying the same.

Skip count in 3's in a variety of ways, forwards, backwards and from different starting points.

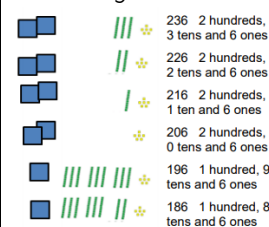
Count on to find complements to benchmark numbers within 100 e.g.

Drawing out complements to benchmark numbers.

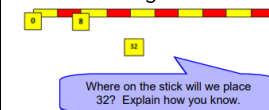


I know there is 7 more to 20 and 5 more to 25.

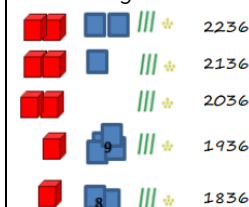
what changes and what doesn't e.g.



Understand the connections between the 3, 4 and 8 times tables and know strategies to be able to work out unknowns e.g.



doesn't e.g.



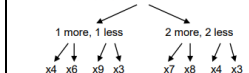
Count on and back in multiples and make counting connections e.g., counting in 6's, 60's, 600's, 0.6's

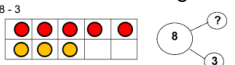
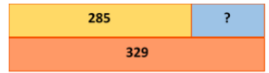
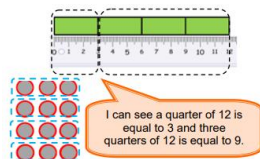
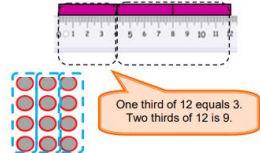
Count in 25's, 50's, 0.1's and 0.001's

Skip count all the times tables and understand the connections between them

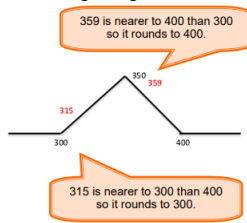
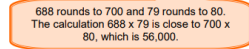
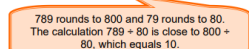
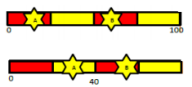
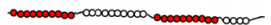
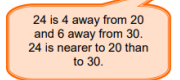


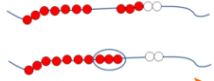
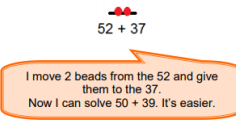
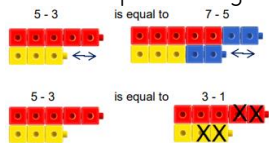
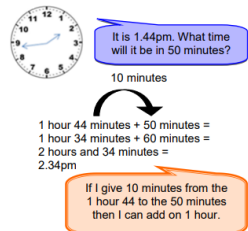
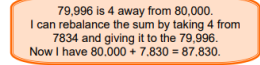
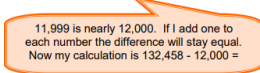
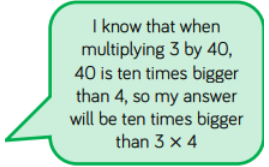
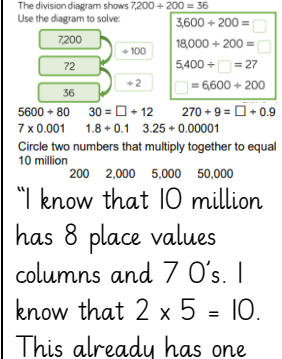
If I know x1, x2, x5, x10, what else can I work out?



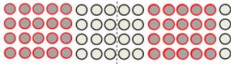
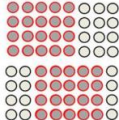
		and from different starting points.				
Reordering and finding complements		Reorder to ensure efficient counting e.g., There are 6 animals.  How many different ways can you sort the animals?	Reorder numbers to find complements when adding three one-digit numbers e.g. $6+7+4$ (add the 6 and 4 first to make 10)	Reorder three or more numbers up to 1000 to find complements e.g., $75+95+25$ can be reordered into $75+25$ to make the benchmark 100, then add 95 to total 195. Or $6+9+4+5+1$ (reorder to make number bonds to 10)	Reorder three or more numbers up to 10,000 to find complements e.g., $800+240+360$ $310+700+30 =$ Reorder three or more numbers involving tenths and hundredths to find complements e.g., $1.5+3+0.5$ $3+1+1$ ($0.5+0.5$)	Reorder three or more numbers to find complements where the arrangement is more complex e.g. $£3.99+£7.80+£2.01$
Applying the inverse		Think addition to solve subtraction with numbers to 10 e.g., 8 - 3  I can see that eight can be split into three and five. I know that five and three are eight. I can see that five more than three is eight. Think addition to solve subtraction with numbers up to 20	Think addition to solve subtraction with numbers up to 100 e.g., Tens Ones - Ones = without regrouping  27 - = 4 4 + = 27 I know 4 and 3 makes 7 so 4 and 23 make 27. Understand the relationship between multiplication and division. In multiplication, understand the interrelationship between	Think addition to solve subtraction with numbers up to 1000 e.g.,  329 - 285 = so 285 + = 329 Think multiplication for division by grouping	Use the inverse to solve missing number equations e.g., $14 + \text{---} = 30$ $30 - 14 = 16$ Make further connections between multiplication and division and fractions  	Use the inverse to solve missing number equations e.g., $14 + \text{---} = 30$ $30 - 14 = 16$ Using BODMAS to practice using inverse equations.

		e.g. $16 - 2 = \square$ $4 \times 3 = 12$ Number in each group 3 Number of groups 4 $12 \div 3 = 4$	multiplier, multiplicand and product. In division, understand the relationship between the dividend, divisor and quotient e.g., $4 \times 3 = 12$	AND sharing e.g., By grouping By sharing		
Rounding			Identify the midpoint between a set of numbers under 1000 (and use this in order to understand	Round to the nearest 10, 100, 1000 and other units such as money and time Use number lines right throughout KS2. Using the rhyme 0-4 touch the floor, 5-9 climb the vine	Round to the nearest 10, 100, 1000, 10,000 and other units such as money, time, decimal numbers, negative numbers Use rounding as an estimation for multiplication and	

				rounding) e.g.,  Use number lines right throughout KS2. Using the rhyme 0-4 touch the floor, 5-9 climb the vine.		division e.g., $688 \times 79 =$  $789 \div 79 =$ 
Estimation			Estimate the distance of numbers from target numbers under 100 e.g.,  Drawing out understanding of the distance of numbers to target numbers.  	Estimate the distance of numbers from target numbers under 1000 to prepare for rounding e.g., 234 is 4 from 230 and 6 from 240. 234 is nearer to 230 than 240	Estimate the distance of numbers from target numbers up to 10,000, including tenths and hundredths to prepare for rounding e.g., 2134 is 34 from 2100 and 66 from 2200. 2134 is nearer to 2100 than 2200.	Estimate the distance of numbers from target numbers up to 1,000,000, including decimal numbers and negative numbers, to prepare for rounding e.g., 20,034 is 4 from 20,030 and 6 from 20,040. 20,034 is nearer to 20,030 than to 20,040.
Rebalancing			Understand the concept of equal sum (that the sum remains equal when the addends are rebalanced in addition)	Use the equal sum concept with numbers up to 1000 as a method to solve addition calculations	Use the equal sum concept with numbers up to 10,000, including units of time and money, as a method to	Apply the equal sum concept to a range of numbers and missing number problems

			with numbers up to 20 e.g. 	e.g., e.g. $52 + 37$  Understand the concept of equal difference (subtracting the same quantity from both subtrahend and minuend maintains the difference) with numbers up to 20 e.g. 	solve addition calculations e.g.,  Use equal difference concept with tenths and simple fractions.	(including units of time and money) e.g., $7834 + 79,996$  Use the equal difference concept with a range of numbers, decimals and fractions e.g. $132,457 - 11,999 =$ 
$X \div$ by powers of 10				Understand the concept of $\times$ and $\div 10$, and use known facts to multiply and divide by 10 and 100 e.g., 	Understand the concept of $\times$ and $\div 10$, and use known facts to multiply and divide by 10, 100 and 1000 e.g., 4000×6 $4 \times 6 = 24 \times 1000 = 24,000$ $240 \div 4$ $24 \div 4 = 6$ $6 \times 10 = 240$	X and $\div$ by powers of 10, including 2 step problems. Includes decimals e.g. 

				10 23 </
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					halve of that between the two groups which will always be 5 units (5 ones or 5 tens)”	
Rearranging					Rearrange the multiplier and multiplicand using knowledge of doubles and halves to make a calculation more manageable e.g., $4 \times 16 =$  Can become 8×8  Doubling the 4x and halving the group of 16 gives me 8×8. The area remains equal. $12 \times 2.5 =$ $16 \times 6 \frac{1}{4} =$ $12 \times 2.5 = 6 \times 5$. I halved the 12 and doubled the 2.5 to make the calculation easier. $16 \times 6 \frac{1}{4} = 8 \times 12 \frac{1}{2} = 4 \times 25 = 100$ I can make this easier for me by doubling and doubling again the $6 \frac{1}{4}$. This means I have to halve and halve again the 16 to maintain the area. Now I get $4 \times 25 = 100$. Apply halving and doubling to fractions e.g., Pupils have already secured conceptual understanding of this rule, for example: $5 \times 4 = 10 \times 2 = 20 \times 1$ Apply this understanding to fractions, for example: $\frac{1}{2} \times \frac{1}{4} =$ If we double the first term and halve the second, we can transform the calculation to: $1 \times \frac{1}{8} = \frac{1}{8}$	

Expectations of recognition at each age group...

EYES

- Recognise the number 0-20
- Know one more and 1 less for number 0-20

- Know the days of the week in order
- Identify 2D shapes: circle, square, triangle, rectangle, hexagon, pentagon
- Identify 3D shapes: sphere, cylinder, cone, cubes, cuboids

Year 1

- Facts within 10 as above and related subtraction facts
- Know o'clock and half past times
- Know the seasons and months in order
- Identify 3D shapes: pyramids, square based pyramids

Year 2

- Facts within 20 as above and related subtraction facts
- Doubles and halves of numbers to 20
- Multiplication and division facts 2, 5 and 10 x tables
- Multiplication facts for 3 x tables (although this is not covered in more depth until year 3, there have been questions on year 2 SATs covering counting in 3's)
- Know quarter past and quarter to times, as well as intervals of 5
- Number of minutes in an hour; number of hours in a day
- Coin recognition up to £2 and note recognition (Know 100p=£1)
- Identify 2D shapes: quadrilaterals, regular and irregular polygons and identify 3D shapes: cuboids, prisms

Year 3

- Sums and differences between pairs of numbers which are multiples of 10 and 100
- Doubles and halves of multiples of 10 or 100
- Complements of fractions with the same denominator that make 1 e.g. $\frac{3}{7} + \frac{4}{7} = 1$
- x 3, x 4, x 8 facts including division facts
- Number of seconds in a minute, number of minutes in an hour, number of hours in a day, number of days in a week
- Number of days in each month and in a year including a leap year

- Recognise right angles, parallel and perpendicular lines and horizontal and vertical lines

Year 4

- Review addition and subtraction facts within 20, ensure application to 10, 100 and 1000 ($6 + 3$, $60 + 30$, $600 + 300$, $6000 + 3000$)
- Doubles and halves of multiples of 10, 100 or 1000 ($6 + 6$, $60 + 60$, $600 + 600$, $6000 + 6000$)
- All multiplication and division facts to 12×12
- Division and multiplication by 10 and 100
- Conversion of kilometres to metres, hours to minutes, years to months, weeks to days
- Understand tenths and hundredths that make 1
- Read roman numerals to 100
- Know right angles = 90 degrees and types of triangle (isosceles, equilateral, scalene)

Year 5

- Identify prime numbers up to 20 (2, 3, 5, 7, 11, 13, 17, 19)
- Recall metric conversions (1 kilogram = 1000 grams, 1 kilometre = 1000 metres, 1 metre = 100 centimetres, 1 metre = 1000 millimetres, 1 centimetre = 10 millimetres 1 litre = 1000 millilitres)
- Recall square numbers up to 12 squared and their square roots
- Convert between decimals and fractions eg. $1/10=0.1$, $1/4=0.25$
- Read Roman numerals to 1000
- Know angles on a straight line = 180 degrees, know angles in a triangle = 180 degrees, know angles around a point = 360 degrees

Year 6

- Multiplication and division by zero and one facts
- Convert between decimals, fractions and percentages
- Identify prime numbers up to 50 (2, 3, 5, 7, 11, 13, 17, 19, 23, 27, 29, 31, 37, 41, 43, 47)
- Illustrate and name parts of a circle, including radius, diameter and circumference and know that the diameter is twice the radius.

1/2	0.5	50%
1/4	0.25	25%
3/4	0.75	75%
1/5	0.2	20%
1/10	0.1	10%
2/5	0.4	40%
1/100	0.01	1%
9/100	0.09	9%
21/100	0.21	21%
1/20	0.05	5%